Chem 121
Homework #4

1. A square matrix with zero determinant is singular.

a) If A is singular, show that there is at least one column vector $\vec{v}$ such that

$$A \cdot \vec{v} = \vec{0} \quad \text{and} \quad |\vec{v}| \neq 0$$

b) If there is a vector $\vec{v} \neq \vec{0}$ such that

$$A \cdot \vec{v} = \vec{0}$$

show that A is singular, i.e. has zero determinant.

Note that this means that an operator with a zero eigenvalue has no inverse!

2. Two Hermitian matrices A and B have the same eigenvalues. Show that A and B are related by a similarity transform.

3. Find the eigenvalues and eigenvectors of

$$\begin{pmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{pmatrix}$$

Are the eigenvalues degenerate for any value of ε?
Are the eigenvectors orthogonal for all ε?

4. A is 2x2 and orthogonal. Find the most general form of

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Compare your result to a 2x2 <u>rotation</u> matrix? What can you conclude?